LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2023**

PMT 2504 – COMPLEX ANALYSIS

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Answer all Questions. All questions carry equal marks.

1. a. Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} dz = 2\pi$ if $|z| < 1$.

OR

b. State and prove Maximum Modulus theorem. (5)

c. Prove that any differentiable complex valued function defined on an open set is analytic.

OR

d. State and prove the homotopic version of Cauchy's theorem. (15)

2. a. Prove that a function $f: [a, b] \to \mathcal{R}$ is convex if and only if the set $A = \{(x, y): a \leq x \leq b \; and \; f(x) \leq y\}$ is a convex set.

OR

b. State and prove Morera's theorem. (5)

c. State and prove Hadamard's three circles theorem.

OR

d. If $f: D \to D$ is a one-one analytic map of D onto itself and f (a) = 0, then prove that there is a complex number c with $|c| = 1$ such that $f = c\varphi_a$ where $\varphi_a(z) = \frac{z-a}{1-\bar{a}z}, |z| < 1$. (15)

3. a. Prove that $sin\pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$.

OR

b. State and prove Gauss's formula. (5)

c. State and prove Weierstrass factorization theorem.

OR

d. State and prove Riemann mapping theorem. (15)

4. a. Find the order of $exp(e^z)$ and $\exp(z^n)$.

OR

b. State and prove Jensen's formula. (5)

c. If f is an entire function of finite genus μ, then prove that f is of finite order $\lambda \leq \mu + 1$.

OR

d. If f is an entire function of finite order λ, then prove that it has finite genus μ and $\mu \leq \lambda$. (15)

5. a. Prove that the sum of residues of an elliptic function is zero.

OR

b. Prove that a non-constant elliptic function has equal number of poles and zeros. (5)

c. Prove that $\wp(2z) = \frac{1}{4}\left(\frac{\wp''(z)}{\wp'(z)}\right)^2 - 2\wp(z)$ and $\wp'(z) = -\frac{\sigma(2z)}{\sigma(z)^4}$, where $\wp(z)$ is Weierstrass $\wp$ function and $\sigma(z)$is sigma function.

OR

d. Prove the following:

1. $\varsigma'(z) = -\wp(z)$, where $\varsigma(z)$, weierstrass zeta function and $\wp(z)$, Weierstrass $\wp$ function
2. $\varsigma\,(z + w_1) = \varsigma(z) + n_1$ and $\varsigma\,(z + w_2) = \varsigma(z) + n_2$ where n_1 and n_2 are constants.
3. $\sigma(z + w_1) = -\sigma(z)e^{n_1\left(z+\frac{w_1}{2}\right)}$ and $\sigma(z + w_2) = -\sigma(z)e^{n_2\left(z+\frac{w_2}{2}\right)}$ where w_1 and w_2 are periods of Weierstrass $\wp$ function $\wp(z)$ and $\sigma(z)$, sigma function. (15)

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